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Shell Effects in the First Sound Velocity of an Ultracold Fermi Gas

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Abstract We investigate the first sound of a normal dilute and ultracold two-component Fermi gas in a harmonic microtube, i.e. a cylinder with harmonic transverse radial confinement in the length-scale of microns. We show that the velocity of the sound that propagates along the axial direction strongly depends on the dimensionality of the system. In particular, we predict that the first-sound velocity display shell effects: by increasing the density, that is by inducing the crossover from one-dimension to three-dimensions, the first-sound velocity shows jumps in correspondence with the filling of harmonic modes. The experimental achievability of these effects is discussed by considering ^{40}K atoms.

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1 Introduction

Macroscopic effects of quantum statistics are observable at ultra-low temperatures with bosonic vapors of alkali-metal atoms[1,2]. In the last years quantum degeneracy has been achieved also with fermionic atoms[3,4,5,6,7]. It is important to observe that the role of dimensionality in these degenerate gases has been experimentally studied until now only with bosons[8,9,10]. For fermions, it has been predicted that a reduced dimensionality strongly modifies density profiles[11,12,13,14], collective modes[15] and stability of mixtures[16,17]. Sound velocity has been theoretically investigated in strictly one-dimensional (1D) and 2D configurations with both normal[15,18] and superfluid Fermi gases[19,20,21].

In this paper we analyze the first-sound velocity of a normal Fermi gas in the 1D-3D crossover. We stress that in previous studies[22,23] the sound velocity of a Bose gas was considered in the same dimensional crossover. Here we consider

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a dilute and ultracold Fermi gas with a strong harmonic confinement along two directions and uniform in the other direction. We show that the first-sound velocity strongly depends on the dimensionality of the system. We reproduce the results of Minguzzi *et al.*[15] and of Capuzzi *et al.*[18] as limiting values. In addition, we predict that the sound velocity display observable shell effects. By increasing the density, that is by inducing the crossover from one-dimension to three-dimensions, the sound velocity show jumps in correspondence with the filling of harmonic modes shells.

2 Confined Fermi gas in a harmonic microtube: 1D-3D crossover

Collective sound modes in a normal Fermi system may be classified into two regimes according to whether collisions between atoms are important or not. If the atomic collision time τ_c is much longer than the period (ω^{-1}) of the mode, i.e. if $\omega \tau_c \gg 1$, collisions may be neglected and the mode, called zero sound, describes the propagation of a deformation of the Fermi sphere[24,25,26,27]. If, on the contrary, collisions between atoms are so frequent, and therefore τ_c so small, that $\omega \tau_c \ll 1$, then the system is in the collisional (or hydrodynamic) regime, and the mode is the ordinary (first) sound corresponding to a the propagation of a density fluctuation[24,25,26,27]. Here we analyze the collisional case of a normal Fermi gas in a harmonic microtube, i.e. a cylinder with a harmonic radial confinement with a characteristic length of the order of some microns. We investigate the regime where the temperature T of the gas is well below its Fermi temperature T_F, i.e. we consider the hydrodynamic regime of an otherwise perfect and completely degenerate Fermi gas[28].

Let us consider the Fermi gas confined by a harmonic potential of frequency $\omega_\perp$ in the $x-y$ plane, while it is free to move along the z axis. The external potential is then:

$$U(\mathbf{r}) = \frac{1}{2}m\omega_\perp^2 (x^2+y^2) , \qquad (1)$$

By imposing periodic boundary conditions in a box of length L along z, the single-particle energies are:

$$\varepsilon_{i_x i_y i_z} = \hbar\omega(i_x+i_y+1) + \frac{\hbar^2}{2m}\frac{(2\pi)^2}{L^2}i_z^2 , \qquad (2)$$

where i_x, i_y are natural numbers and i_z is an integer. The total number of particles is given by:

$$N = 2 \sum_{i_x i_y i_z} \Theta(\bar{\mu} - \varepsilon_{i_x i_y i_z}) , \qquad (3)$$

where the factor 2 takes account of spins, $\Theta(x)$ is the Heaviside step function, and $\bar{\mu}$ is the chemical potential (Fermi energy). In a cigar geometry, with $L \gg 2\pi\hbar/\sqrt{2m\bar{\mu}}$, one may define the quasi-continuum variable $k_z = \frac{2\pi}{L}i_z$ and rewrite the number of particles as

$$N = 2 \sum_{i_x i_y} \frac{L}{2\pi} \int dk_z\, \Theta(\bar{\mu} - \varepsilon_{i_x i_y k_z}) . \qquad (4)$$

Since the motion is free along this direction, the 1D density $n_1 = \frac{N}{L}$ does not depend on z, and it can be written as

$$n_1 = \frac{1}{\pi} \sum_{i_x i_y} \int dk_z \Theta(\bar{\mu} - \varepsilon_{i_x i_y k_z}) = \frac{1}{\pi} \sum_{i=0}^{\infty} (i+1) \int dk_z \Theta(\bar{\mu} - \varepsilon_{i k_z}), \qquad (5)$$

where $\varepsilon_{i k_z} = \hbar \omega_{\perp}(i+1) + \hbar^2 k_z^2/(2m)$. The term with $i = 0$ gives the density of a strictly 1D Fermi gas (only the lowest single-particle mode of the harmonic oscillator is occupied), while the terms with $i > 0$ take into account occupation of the excited single-particle modes of the harmonic oscillator. After integration, the density n_1 can be written as

$$n_1 = \sum_{i=0}^{I\left[\frac{\mu}{\hbar \omega_{\perp}}\right]} (i+1) \frac{2^{3/2}}{\pi a_{\perp}} \sqrt{\frac{\mu}{\hbar \omega_{\perp}} - i}, \qquad (6)$$

where $a_{\perp} = \sqrt{\hbar/(m \omega_{\perp})}$ is the characteristic length of the transverse harmonic potential, $\mu = \bar{\mu} - \hbar \omega_{\perp}$ is the chemical potential measured with respect to the ground state energy and $I[x]$ is the integer part of x, so that the summation extends from the ground state to the last higher occupied state, which may be only partially occupied.

In the upper panel of Fig. 1 we plot the chemical potential μ versus the transverse density n_1 obtained by using Eq. (6). The Fermi gas is strictly 1D only for $0 \le n_1 < 2^{3/2}/(\pi a_{\perp})$, i.e. for $0 \le \mu < \hbar \omega_{\perp}$. In this case from Eq. (6) one finds

$$\mu = \frac{\pi^2}{8} \hbar \omega_{\perp} (a_{\perp} n_1)^2, \qquad (7)$$

For $n_1 > 2^{3/2}/(\pi a_{\perp})$, i.e. for $\mu > \hbar \omega_{\perp}$, several single-particle states of the transverse harmonic oscillator are occupied and the gas exhibits the 1D-3D crossover, becoming fully 3D when $n_1 \gg 2^{3/2}/(\pi a_{\perp})$, i.e. when

$$\mu = \left(\frac{15\pi}{2^{7/2}}\right)^{2/5} \hbar \omega_{\perp} (a_{\perp} n_1)^{2/5} \gg \hbar \omega_{\perp} \qquad (8)$$

as obtained from (6).

Since the Fermi temperature T_F is related to the chemical potential by the simple equation $k_B T_F = \mu$, where k_B is Boltzmann's constant, our results are valid if the temperature T of the system satisfies the condition $T \ll \mu/k_B$. For instance, in the case of ^{40}K atoms, setting $a_{\perp} = 0.25~\mu$m, we find the harmonic transverse frequency $\omega_{\perp} \simeq 25$ kHz, which is well below the maximal confining transverse frequency obtained with permanent-magnetic atoms chips fo the study of long and thin atom clouds[29]. The Fermi temperature of a strictly 1D gas of ^{40}K atoms is then $T_F \simeq 130$ nKelvin.

To analyze a sound wave that travels in the axial direction it is important to determine the collision time τ_c of the gas. According to Bruun et al.[28] and Gupta et al.[30], if the local Fermi surface is not strongly deformed then $\tau_c = \tau_0(T/T_F)^2$, where $\tau_0 = 1/(n\sigma v_F)$ with σ the scattering cross-section and n the 3D density. Instead, if the local Fermi surface is strongly deformed as in our case, then the
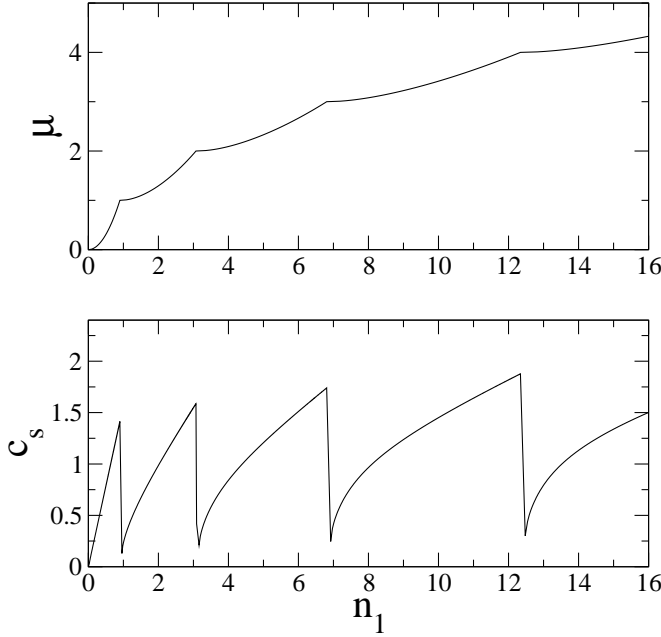
Fig. 1 Chemical potential μ and first-sound velocity c_s as a function of the axial density n_1 for an axially uniform Fermi gas with a radial harmonic confinement of frequency $\omega_\perp$. Chemical potential in units of $\hbar\omega_\perp$, sound velocity in units of $a_\perp\omega_\perp$, and density in units of $1/a_\perp$, where $a_\perp = \sqrt{\hbar/(m\omega_\perp)}$.

collision time is simply given by $\tau_c = \tau_0$.[30] It is easy to find that in our quasi-1D geometry $\tau_0\omega_\perp \simeq 1/(n_1 a_s)^2$, where a_F is the s-wave scattering length between fermions with opposite spins. As said above, the wave propagates in the collisionless regime if $\omega\tau_c \gg 1$ and in the collisional regime if $\omega\tau_c \ll 1$. The collision time ^{40}K atoms under a transverse harmonic confinement of frequency $\omega_\perp \sim 25$ kHz, is $\tau_c \sim 10^{-2}$ sec and therefore the hydrodynamic regimes will be obeyed by waves with frequencies up to $\omega \simeq 10^2$ Hz.

In the collisional regime, by using the thermodynamic formula which relates the compressibility to the chemical potential[24,25,26,27] we immediately find that the axial first sound velocity c_s of the Fermi gas is

$$c_s = \sqrt{\frac{n_1}{m}\frac{\partial\mu}{\partial n_1}} \ . \tag{9}$$

This formula, supplemented by Eq. (6), enables us to determine the behavior of c_s versus n_1, as shown in the lower panel of Fig. 1. The figure clearly displays shell effects, namely jumps of the first sound velocity c_s when the atomic fermions occupy a new axial harmonic mode. These shell effects could be tested experimentally. In fact, due to the large anisotropy, transverse modes are decoupled from the axial sound modes.[31]

As previously stressed the Fermi gas is strictly 1D only for $0 \leq n_1 < 2^{3/2}/(\pi a_\perp)$, i.e. for $0 \leq \mu < \hbar\omega_\perp$. In this case from Eqs. (6) and (9) one finds

$$c_s = \frac{\pi}{2} a_\perp \omega_\perp a_\perp n_1 . \tag{10}$$

Note that the asymptotic formula (10) is exactly that discussed by Minguzzi *et al.*[15] Instead, for $n_1 \gg 2^{3/2}/(\pi a_\perp)$, i.e. for $\mu \gg \hbar\omega_\perp$, the Fermi gas becomes 3D and Eqs. (6) and (9) one gets

$$c_s = \sqrt{\frac{2}{5}} \left(\frac{15\pi}{2^{7/2}}\right)^{1/5} a_\perp \omega_\perp \left(a_\perp n_1\right)^{1/5} . \tag{11}$$

The asymptotic result (11) coincides with that found by Capuzzi *et al.*[18] We stress that in strictly one-dimensional Fermi system the hydrodynamical approach gives the full spectrum of collective density fluctuations and the zero-sound velocity coincides with the first sound velocity[32].

To determine the conditions under which the first sound can be detected with a quasi-1D two-component Fermi gas, we consider again ^{40}K atoms with scattering length $a_F \simeq 150 \cdot 10^{-10}$ m. In a cylindrical configuration with $L = 1$ mm and $a_\perp = 0.25~\mu$m we get the collision time $\tau_c \simeq 1.1 \cdot 10^{-2}$ sec, and the sound velocity $c_s \simeq a_\perp \omega_\perp = 6$ mm/sec. The Fermi system is strictly 1D if the axial density n_1 does not exceed the value $n_1 \simeq 4$ atoms/μm^{-1}. The characteristic wave length of collision is $\lambda_0 = c_s \tau_c \simeq 69~\mu$m. The condition for collisional regime is that the wave length $\lambda = c_s 2\pi/\omega$ of the sound wave is larger than λ_0. Thus, pertubing the axially uniform Fermi gas with a blu-detuned laser beam with a width of, for instance, $\simeq 150~\mu$m one produces two counter-propagating axial waves moving at the first-sound velocity c_s.

3 Conclusions

We have shown that the first-sound velocity gives a clear signature of the dimensional crossover of a two-component normal Fermi gas. Our calculations suggest that the dimensional crossover induces shell effects, which can be detected as jumps in the first-sound velocity. We have discussed the experimental achievability of first sound by using gases of ^{40}K atoms, finding that the collisional regime requires severe geometric and thermodynamical constraints. Finally, it is relevant to stress that in a Fermi system the sound propagates also in the collisionless regime due to mean-field effects. In this regime the velocity of (zero) sound can be determined by using the Boltzmann-Landau-Vlasov kinetic equation of the phase-space Wigner distribution function. This important issue will be considered elsewhere.

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